Course description and syllabus

General information

Instructors: Prof. Chris Wendl (lectures)
HU Institut für Mathematik (Rudower Chaussee 25), room 1.301
`wendl@math.hu-berlin.de`

Problem class (Übung) instructor TBA

Website: `http://www.mathematik.hu-berlin.de/~wendl/Winter2020/FunkAna/`

Lectures: Tuesdays 15:00–17:00
Thursdays 13:00–15:00

Problem classes: Thursdays 15:00–17:00

The course will be conducted online via Zoom. Links for the Zoom meetings will be made available on the moodle for this course shortly before the start of the semester.

English: This course is counted as a *BMS Basic Course*, thus it is offered in English unless everyone would like to hear it in German. This will be decided in the first lecture. If you definitely want to hear the course in English but cannot make it on time to the first lecture, please contact the instructor in advance.

Prerequisites: The contents of the HU's courses *Analysis I–III* and *Lineare Algebra und Analytische Geometrie I–II*, as well as the analytical part of the course *Algebra und Funktionentheorie* (i.e. basic complex analysis).

All students should in particular be familiar with the fundamentals of measure theory (including Fubini's theorem and the completeness of the L^p spaces).

Course description

This is a course on *linear* functional analysis, which can be defined as the study of continuous linear maps between infinite-dimensional topological vector spaces (mainly Banach and Hilbert spaces). The most important examples of such infinite-dimensional vector spaces are function spaces, which often arise in applications e.g. as solution spaces for partial differential equations. The contents of this course should therefore be seen as essential preparation for any course (either in analysis, applied mathematics, differential geometry or mathematical physics) dealing with PDEs.

Syllabus

The following week-by-week schedule is preliminary and subject to change.

1. Basic notions from point-set topology, topological vector spaces, locally convex vector spaces, Fréchet, Banach and Hilbert spaces, examples

2. Continuous/bounded linear operators between Banach spaces, the operator norm, dual spaces, Zorn's lemma, Hamel bases
3. Basic results on Hilbert spaces: the Riesz representation theorem, orthonormal bases, orthogonal projections
4. Properties of the L^p spaces on $\mathbb{R}^n$: duality of L^p and L^q for $\frac{1}{p} + \frac{1}{q} = 1$, convolutions and Young's inequality, approximation by smooth functions
5. Separability of L^p, weak convergence, the Banach-Alaoglu theorem, absolute continuity and the fundamental theorem of calculus
6. The Fourier transform on Schwartz space and $L^2(\mathbb{R}^n)$
7. Periodic functions and Fourier series on $L^2(\mathbb{T}^n)$
8. The Sobolev spaces $H^k(\mathbb{R}^n)$ and $H^k(\mathbb{T}^n)$, distributions (generalized functions)
9. The Baire category theorem and Hahn-Banach theorem
10. The open mapping theorem, closed subspaces with closed complements
11. Compact operators and Fredholm operators
12. The spectrum of a bounded linear operator on a Hilbert space, polar decomposition
13. Spectral theory for bounded operators
14. Unbounded self-adjoint operators and spectral theory

A possible additional topic, if time remains: differential calculus for nonlinear maps between Banach spaces

Literature

The course will not follow any particular book, but the following textbooks are highly recommended, especially the book by Reed and Simon.

- Reed and Simon, *Methods of Modern Mathematical Physics I, Functional Analysis*, revised and enlarged edition, Elsevier 2011
  (online access available via the HU library)
- Bühler and Salamon, *Functional Analysis*, AMS 2018
  (preprint version available freely on Salamon's homepage:
  `https://people.math.ethz.ch/~salamon/PREPRINTS/funcana-ams.pdf`)
- Conway, *A Course in Functional Analysis*, Springer 1985
  (online access available via the HU library)

For topics that actually belong to measure theory, such as the properties of the L^p spaces and distributions, we also recommend:

- Lieb and Loss, *Analysis*, 2nd edition, AMS 2001
  (available in the HU library, Freihandbestand)

Exam and problem sets

Grades in this course will be determined by a 30-minute oral exam soon after the end of the semester (with a resit option shortly before the beginning of the following semester). In the exam, you will need to be able to write down the main definitions in the course, discuss their meaning and significance (with reference to examples where appropriate), and describe the most important applications of the major theorems and the main ideas behind their proofs.

There will be ungraded **problem sets** handed out every Thursday and discussed in the problem class on the following Thursday.

There will also be one graded assignment midway through the semester, a so-called **take-home midterm**, which you will have two weeks to work on. The midterm is voluntary, but achieving a score of 75% or better can boost your final exam grade by one notch, i.e.

$$\text{Midterm} \geq 75\% = (2{,}0 \to 1{,}7 \text{ oder } 1{,}7 \to 1{,}3 \text{ etc.})$$