h-principle seminar — TALK 6 ADDENDUM — 28/2/2013 — J. NORDSTRÖM

microextension & coclosed G_2-structures

G_2 = stabiliser of $\varphi_0 \in \Lambda^3 \mathbb{R}^7$

open orbit $\Lambda^3_+ \mathbb{R}^7$. ~~[illegible]~~ $G_2 \subset SO(7) \rightsquigarrow$

$\varphi \in \Lambda^3_+ \mathbb{R}^3$ induces a metric $\rightsquigarrow$ nonlinear map $\varphi \longmapsto \psi = *\varphi \in \Lambda^4_+ \mathbb{R}^7$

Stab $\psi_0 = G_2 \times \mathbb{Z}_2$.

G_2-str. on $M^7 \longleftrightarrow \varphi \in \mathrm{Sec}\,\Lambda^3_+$

$\longleftrightarrow \psi \in \mathrm{Sec}\,\Lambda^4_+$ + orientation.

closed if $d\varphi = 0$, coclosed if $d\psi = 0$, torsion free if both. ($\Leftrightarrow \mathrm{Hol} \subseteq G_2$).

claim: parametric h-principle for coclosed G_2-strs:
(obvious from earlier thms if M open)

Let $a \in H^4_{dR}(M^7)$. Then $\mathrm{Clo}_a \Lambda^4_+ \simeq \mathrm{Sec}\,\Lambda^4_+$

base case: $\chi_0 = \varphi_0 \wedge dt + \psi_0 \in \Lambda^4 \mathbb{R}^8$ has Stab $\cong Spin(7)$

transitive action on $S^7 \longrightarrow \chi_0 \in \mathcal{R}$, where

$\mathcal{R} := \{ \chi \in \Lambda^4 \mathbb{R}^8 \mid \chi|_H \in \Lambda^4_+ H \;\; \forall \text{ 7-dimensional } H \subset \mathbb{R}^8 \}$
is open & nonempty.

over $V = M \times (-1,1)$, $\mathcal{R}(V) \subseteq \Lambda^4 T^* V$
is open & Diff-inv't.

$\varphi \in \mathrm{Sec}\,\Lambda^4_+(M) \rightsquigarrow \chi = \varphi \wedge dt + \psi \in \mathrm{Sec}\,\mathcal{R}(V)$

$\rightsquigarrow$ homotopic $\chi' \in \mathrm{Clo}_a \mathcal{R}(V)$

$\rightsquigarrow \chi'|_{M \times \{0\}}$ is homotopic to φ.

parametric case: $\psi_1, \psi_2 \in \mathrm{Clo}_a \Lambda^4_+(M)$, set

$\chi := \varphi_i \wedge dt + \psi_i + t\, d\varphi_i$

Then $d\chi_i = 0$, $\chi_i \in \mathrm{Sec}\,\mathcal{R}$ over $M \times (-\varepsilon, \varepsilon)$