Berlin-Leipzig Day Mathematical Statistics

Berlin, January 11th, 2019

Abstracts
Algebraic Learning of Gaussian Mixtures

Carlos Amendola
Technische Universität München

We study the problem of estimation of parameters of statistical models given by a mixture of Gaussian distributions. We compare the prominent techniques of maximum likelihood estimation and method of moments from an algebraic point of view. Following up on Pearson’s classical work from 1894, we apply current tools from computational algebra to recover the parameters from the moments, and we ask in general if a Gaussian mixture can be identified from all the moments up to certain order.

Introduction to the Wrapping Hull and its Applications in Set Estimation

Nicolai Baldin
University of Cambridge

After introducing classical and modern approaches to set estimation, we will dive into Poisson point processes and their properties. We will describe how augmenting a classical modelling assumption, one can not only prove classical results with ease but also unlock new directions for research. We will introduce the so-called wrapping hull, a generalisation of the convex hull, and explain how it can be efficiently used to estimate a set and its functionals. The talk will rest upon an interplay between probabilistic and geometric arguments. We will finalise the talk with a list of open questions in (stochastic) geometry, point processes and statistics for further research.

Tell me how Many Modes Does the Gaussian Mixture Have ...?

Christian Haase
Freie Universität Berlin

Gaussian mixture models are widely used in Statistics. A fundamental aspect of these distributions is the study of the local maxima of the density,
or modes. In particular, it is not known how many modes a mixture of $k$ Gaussians in $d$ dimensions can have. We give improved lower bounds and the first upper bound on the maximum number of modes, provided it is finite. (joint work with Carlos Améndola and Alexander Engström)

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**Uniform Sampling from Varieties**

Orlando Marigliano  
Max-Planck-Institut für Mathematik in den Naturwissenschaften

A (real) variety is a subset of $\mathbb{R}^n$ defined as the set of common zeros of a collection of polynomials. What does it mean to choose one point at random from a variety? Assuming it has finite volume, there exists the uniform probability distribution on it, so in that case the most intuitive answer would be that a random point on the variety is a uniformly distributed one. This talk presents the problem of sampling from the uniform distribution on a variety, given just its defining polynomials. I discuss why this is a desirable thing to do, and explain how an idea from algebraic geometry gives us a way to do it, using examples from a computer implementation of this method.

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**Nonparametric Testing of Laws of Stochastic Processes**

Harald Oberhauser  
University of Oxford

The moments of a vector-valued random variable are a fundamental tool in statistics. I will talk about ideas from stochastic analysis that provide “moment” of path-valued random variables - that is, stochastic processes - and whether these “moment” characterize the underlying probability measure on the space of paths. To address this question I combine recent insights from statistical learning with some classic functional analysis and algebra. To demonstrate how this leads to new statistical tools we discuss a two-sample hypothesis test for laws of stochastic processes. Join work with Ilya Chevyrev.
Equivalence Theorems in Experimental Design
Frank Röttger
Otto-von-Guericke-Universität Magdeburg

Among the most important tools in the theory of optimal design are equivalence theorems, especially the famous Kiefer-Wolfowitz equivalence theorem. These theorems reveal a fruitful connection between statistical optimization of experiments and algebraic structures, namely varieties and semi-algebraic sets. This happens because optimality conditions in these theorems are often given as algebraic constraints on the parameters. In special but important situations, optimality regions are non-negative real parts of varieties. Furthermore, the varieties and semi-algebraic sets appearing in equivalence theorems often show a lot of symmetry, which might be useful to simplify the algebraic computation of optimal designs.

Central Limit Theorem for Bures-Wasserstein Barycenters
Alexandra Suvorikova
Universität Potsdam

In this work we introduce the concept of Bures-Wasserstein barycenter $Q_*$, that is essentially a Fréchet mean of some distribution $\mathcal{P}$ supported on a subspace of positive semi-definite Hermitian operators $\mathbb{H}_+(d)$. We allow a barycenter to be restricted to some affine subspace of $\mathbb{H}_+(d)$ and provide conditions ensuring its existence and uniqueness. We also investigate convergence and concentration properties of an empirical counterpart of $Q_*$ in both Frobenius norm and Bures-Wasserstein distance, and explain, how obtained results are connected to optimal transportation theory and can be applied to statistical inference in quantum mechanics.

Information Inequalities under Group Action with Applications to PCA
Martin Wahl
Humboldt-Universität zu Berlin
In this talk, we derive non-asymptotic upper and lower bounds for the estimation of the eigenspaces of a high-dimensional covariance operator. The upper bounds rely on a refinement of standard perturbation theory, taking into account the particular structure of the underlying perturbation problem. The lower bound is based on an information inequality under group action, which lies between the Cramér-Rao inequality and the van Trees inequality. We discuss the issue of optimality for three classes of eigenvalue behaviour: exponential decay, polynomial decay, and the spiked covariance model. This is partly based on Reiβ and Wahl (2016) and Jirak and Wahl (2018).